Effect of Finite-Size Probe Volume upon Laser Doppler Anemometer Measurements

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The biasing of an ensemble of individual realization laser Doppler anemometer data caused by both spatial and temporal variation of the velocity in the probe volume has been analyzed. The analysis shows that the time- and space-average velocity through the probe volume can be calculated with a weighted ensemble of data. Likewise, the root-mean-square velocity in the probe volume can also be calculated with a weighted average. However, the velocity variance in the data ensemble originates from three sources: velocity fluctuations caused by turbulence at the center of the probe volume; spatial variations in the mean velocity across the probe volume; and spatial variations in velocity fluctuations across the probe volume. Since the root-mean-square velocity at the center of probe volume is generally the parameter of interest, the other two sources of velocity variance must be subtracted from the measured value. The results of the analysis are used to reduce LDA data taken in the viscous sublayer of a channel flow of water. Without correction, the mean velocity estimates can be as much as 10% too high while the root-mean-square estimates are as much as 100% high. The proposed correction techniques brings the LDA data into good agreement with previous hot-film data.

Nomenclature

A = cross sectional area of probe volume normal to the flow direction
a_j = number of realizations from the j^{th} slice of the probe volume
b = total number of slices in probe volume
c = constant related to seed density and length of time data was taken
K = constant
i,j,k = indices for summation
N = total number of velocity realizations
n = number of velocity realizations
P = probability density function
Re = Reynolds Number based upon average velocity and channel width
S = time-average, fluid velocity gradient
T = time interval of integration or measurement
t = time
U = streamwise velocity
U_B = biased mean velocity, ensemble average of individual realizations
U_c = corrected mean velocity, weighted average of individual realizations
U_m = spatially averaged fluid velocity
U_0 = velocity at the center of the probe volume
U_τ = shear velocity
u' = root-mean-square velocity
u'_B = biased root-mean-square velocity
u'_m = root-mean-square velocity averaged over the entire probe volume
u'_0 = root-mean-square velocity at the center of the probe volume
V_{jk} = velocity vector
w = width of the probe volume
y = spatial coordinate normal to the wall
y_0 = center of the probe volume
y^+ = nondimensional distance from the wall
z = spatial coordinate normal to the flow and parallel to the wall
γ = geometric factor
η = distance in the y-direction from the center-line of the probe volume
ν = kinematic viscosity
ω = weighting function
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Introduction

WITH the recent development of laser Doppler anemometers, LDA, the potential for making very accurate velocity measurements in flowfields has increased substantially. The primary features which yield this new potential are the absence of a physical probe in the flowfield and the instrument's linear response to a single component of the fluid velocity. There are however, new problems such as the ambiguities that result from several scattering particles being in the probe volume at the same time, intermittant and uncontrollable loss of signal, the ability of scattering particles to follow fluctuating and rapidly accelerating flows, and the finite extent of the probe volume. All but the latter of these difficulties can be circumvented by carefully controlling the

Received Oct. 14, 1975; revision received March 19, 1976. The authors would like to express their appreciation to D. McLaughlin for his critical review of the manuscript. The School of Mechanical and Aerospace Engineering of the Oklahoma State University supported this work.
Index category: Lasers; Nozzle and Channel Flow
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size and number density of the scattering particles so that the output Doppler signal is due to a single particle which is following the flow.[1] The resulting mode of operation is called the individual realization or counting mode and the dual-scatter or fringe-anemometer optical system is commonly used.[2] The output signal consists of frequency bursts resulting from individual particles crossing the fringe field. Various techniques are used to measure the frequency of these bursts and construct an ensemble of velocity realizations over a period of time. However, the resulting ensemble is biased, and accurate estimates of the statistical properties of the fluid velocity field cannot be made unless the analysis includes corrections that compensate for the sampling bias.

Biasing of the ensemble arises because, for a constant seed density, the probability of a velocity realization is proportional to the velocity of the fluid in the probe volume. The biasing of the ensemble caused by velocity fluctuations in time has been analyzed and a correction proposed by McLaughlin and Tiederman.[3] Both Kried[4] and Mayo[5] have considered the bias resulting from a mean velocity gradient within a finite probe volume. Mayo restricted his analysis to a simple square probe volume cross section and to the effect of the velocity gradient upon the calculation of the mean and the "apparent" root-mean-square velocity in a laminar flow. Kried considered the effect upon the mean velocity for a more general ellipsoidal probe volume in both laminar and turbulent flow. However, in the turbulent case, Kried separately analyzed the biasing resulting from spatial variations and that resulting from temporal variations. This leads to an incorrect result because the physical reason for the bias is the same for both a spatial and a temporal variation; specifically, the probability of a velocity realization is simply proportional to the local instantaneous velocity and therefore the temporal and spatial effects cannot be separated.

The purpose of this paper is to analyze the biasing of an ensemble of individual realizations caused by a velocity gradient in a turbulent flow and thereby to determine the effect of a finite LDA probe volume upon the calculation of the mean and root-mean-square velocity from an ensemble of individual realizations in a turbulent flow. The bulk of the analysis will be restricted to a probe volume that has a rectangular cross section and to a velocity field where both the time-average fluid velocity and the root-mean-square fluid velocity vary linearly with distance from the wall. There are several reasons for these simplifying assumptions. First, Kried has shown how to analyze complex probe volume geometries. More importantly, the rectangular cross-section is a good model for the laser Doppler anemometer probe volume that was used by Karpuk and Tiederman[6] to measure mean and fluctuation velocities within the viscous sublayer of a turbulent, two-dimensional channel flow. The analysis will be used to reduce these sublayer data and thus demonstrate the magnitude of the biasing with actual data. The assumptions of linear spatial variations are obviously appropriate for this application.

Viscous sublayer data provide an excellent test for the analysis because the high velocity gradient region is small when compared to the size of anemometer probe volumes. A typical hot-wire probe for these applications has a diameter of 5 μm. It is not easy to pick a typical value for the critical dimension of a laser anemometer; however, Karpuk and Tiederman[6] developed an optical system specifically for viscous sublayer measurements in which the probe volume was effectively a 244 μm long cylinder with a diameter of 61 μ m. As with the single component hot-wire, the probe volume was oriented so that the axis of the cylinder was parallel to the wall and normal to the streamwise velocity. Although this laser anemometer probe volume has a diameter which is 12 times the diameter of a standard hot wire, it is several times smaller than LDA probe volumes used previously.[7] This comparison shows why the finite size of the probe volume is a source of concern. It is certainly possible to make further reductions in the size of laser anemometer probe volumes. However, the size will always be finite and here, therefore, we will evaluate the effect upon the mean and root-mean-square of the streamwise velocity component as the relative size of the critical probe volume dimension as compared to the velocity gradient is varied.

In order to demonstrate both the existance and the nature of sampling bias, a laminar analysis will be presented first. In this section, a technique for reducing the data that removes the bias will also be introduced. A similar technique will then be used to develop equations that yield good estimates of the time-average fluid velocity and the root-mean-square fluid velocity from an ensemble of individual realizations in turbulent flow. These turbulent flow equations will then be used to analyze the ensembles, measured in the viscous sublayer of a turbulent channel flow of water. As the analysis of this sublayer data will show, errors as large as 10% can be made if standard statistical techniques are used to estimate the mean velocity, while the biased root-mean-square velocity can be as much as 100% high. The proposed correction technique brings the fluctuation data of Karpuk and Tiederman into good agreement with previous hot-film measurements of Eckelmann and Reichardt.[8]

Laminar Flow Analysis

The primary objective of this laminar flow analysis is to show how the fluid velocity at the center of the probe volume, U_0, can be calculated from an ensemble of individual realizations when there is a velocity gradient across the probe volume. Implicit in this objective is the suggestion that a simple unweighted average of the ensemble, U_B, will not be equal to U_0. This latter result has been derived by both Kried[4] and Mayo.[5]

The unweighted average velocity of a LDA ensemble of velocity realizations is given by

$$U_B = (1/N) \sum_{i=1}^{N} U_i \tag{1}$$

This summation can be written in an integral form that can be evaluated when the velocity profile is known and when the scattering particles (seed) are, on the average, uniformly distributed throughout the fluid. In this case, the number of realizations, Δn, contributed to the ensemble from a small element of the probe volume's cross sectional area, ΔA, is proportional to the velocity at that point and to the probability, $((A)$, that the light scattered from a seed particle in that small area will be detected. That is

$$\Delta n = cU(A)P(A)\Delta A \tag{2}$$

Here c is a constant that is proportional to the number density of the seed and the length of time over which the measurements are recorded. $P(A)$ is a function of variations in the incident light intensity across the probe volumes, the size distribution of the seed and the configuration of the LDA optics. Thus, for a long period of time, Eq. (1) becomes

$$U_B = \frac{\iint U(A)[cU(A)P(A)]\mathrm{d}A}{\iint cU(A)P(A)\mathrm{d}A} \tag{3}$$

If a velocity field and a probe volume shape are known or assumed, then Eq. (3) can be integrated. For a two-dimensional velocity field that varies linearly with y across the probe volume

$$U(A) = U(y,z) = U_0 + S\eta \tag{4}$$

where

$$\eta = y - y_0 \tag{5}$$

S is the velocity gradient and $y=y_0$ is a line through the center of the probe volume (see Fig. 1). Using Eqs. (4) and (5), Eq. (3) can be rewritten as

$$U_B=\frac{U_0^2\iint P(\eta,z)\,\mathrm{d}\eta\mathrm{d}z+2U_0S\iint\eta P(\eta,z)\,\mathrm{d}\eta\mathrm{d}z+S^2\iint\eta^2P(\eta,z)\,\mathrm{d}\eta\mathrm{d}z}{U_0\iint P(\eta,z)\,\mathrm{d}\eta\mathrm{d}z+S\iint\eta P(\eta,z)\,\mathrm{d}\eta\mathrm{d}z} \tag{6}$$

Equation (6) can be simplified by noting that

$$\iint P(\eta,z)\,\mathrm{d}\eta\mathrm{d}z=1 \tag{7}$$

and by assuming $P(\eta,z)$ is symmetric about the center line of the probe volume ($\eta=0$) so that

$$\iint\eta P(\eta,z)\,\mathrm{d}\eta\mathrm{d}z=0 \tag{8}$$

Equation (6) becomes

$$U_B=U_0+\frac{S^2}{U_0}\int\int\eta^2P(\eta,z)\,\mathrm{d}\eta\mathrm{d}z \tag{9}$$

Thus, the unweighted ensemble average of LDA data is biased by the second moment of the probe volume's spatial probability density function.

If the cross sectional area of the probe volume is a rectangle of width w, and if $P(\eta,z)$ is assumed to be constant, Eq. (9) integrates to yield

$$U_B=U_0+S^2w^2/(12U_0) \tag{10}$$

If an elliptically shaped cross sectional area with a width w is assumed, Eq. (9) yields

$$U_B=U_0+S^2w^2/(16U_0) \tag{11}$$

These results are consistent with those obtained previously by Mayo and Kried. Besides showing that $U_B\neq U_0$, Eqs. (10) and (11) also show that the biasing is greatest with a rectangular cross section. Physically this may be explained simply by realizing that the rectangular shape will have proportionally more area farther from $\eta=0$.

While Eqs. (10) or (11) could be used to determine U_0 if the velocity gradient and probe volume width are known, a weighted average of the ensemble of LDA data yields U_0 directly. Specifically,

$$U_c=\frac{\sum_{i=1}^{N}(1/U_i)U_i}{\sum_{i=1}^{N}(1/U_i)}=\frac{N}{\sum_{i=1}^{N}(1/U_i)} \tag{12}$$

Equation (12) may be rewritten in integral form using Eq. (2). The result is

$$U_c=\frac{\iint cU(A)P(A)\,\mathrm{d}A}{\iint cP(A)\,\mathrm{d}A} \tag{13}$$

For the linear velocity profile given by Eq. (4), Eq. (13) becomes

$$U_c=\frac{U_0\iint P(\eta,z)\,\mathrm{d}\eta\mathrm{d}z+S\iint\eta P(\eta,z)\,\mathrm{d}\eta\mathrm{d}z}{\iint P(\eta,z)\,\mathrm{d}\eta\mathrm{d}z} \tag{14}$$

and therefore, for a symmetric probability density function,

$$U_c=U_0 \tag{15}$$

The weighting in Eq. (12) is the one-dimensional weighting proposed by McLaughlin and Tiederman[3] to correct for biasing caused by turbulent fluctuations. This weighting is appropriate for correcting the bias caused by velocity graidents in laminar flow because the probability for an individual realization is proportional to the streamwise velocity.

It is also straightforward to construct numerical examples that use Eqs. (2) and (12) and show the same result. The procedure is to divide the probe volume into a number of equal width slices and then calculate the number of realizations in each slice using Eq. (2).

If $P(A)$ is a constant within the probe volume, then Eq. (13) becomes

$$U_c=(1/A)\iint U(A)\,\mathrm{d}A \tag{16}$$

Thus, U_c is equal to the spatially average fluid velocity U_m through the probe volume. Moreover, if the velocity field is also assumed to be only a linear function of y, then

$$U_c=U_m=U_0 \tag{17}$$

An ensemble of individual realization data from a finite probe volume located in a velocity gradient will have a variance or "false turbulence," since the velocity measurements originate from different locations in the probe volume. Mayo[5] has evaluated this false turbulence for the case of a rectangular probe volume, a laminar, linear velocity profile and a constant value of $P(A)$. His result is

$$u'^2=S^2w^2/12 \tag{18}$$

This variance in the laminar data ensemble, which is caused by the change in mean velocity across the probe volume, also exists in turbulent flows, as will be shown later.

Mean Velocity Analysis for Turbulent Flow

The mean velocity analysis for turbulent flow is similar to that for the laminar case. There is one additional complication, namely, a time integration or time average process which must be explicitly done for the turbulent case. Thus, the appropriate average fluid velocity flowing through a probe volume, oriented so as to measure the streamwise component of velocity, is both a temporal and spatial average given by

$$\bar{U}_m=(1/A)\int\int[(1/T)\int_0^T U(A,t)\,\mathrm{d}t]\mathrm{d}A \tag{19}$$

Here, $U(A,t)$ is the instantaneous streamwise velocity, t is time and T is the time interval for the measurement.

If the discussion is limited to a rectangular probe volume in a two-dimensional flowfield such that $\bar{U}_m$ is not a function of z, then Eq. (19) becomes

$$\bar{U}_m=(1/w)\int_{-w/2}^{w/2}[(1/T)\int_0^T U(\eta,t)\,\mathrm{d}t]\mathrm{d}\eta \tag{20}$$

Here, w is the probe volume dimension in the direction of the mean velocity gradient, y. The problem now is to show how $\bar{U}_m$ can be estimated using the intermittent and randomly occuring individual realizations of velocity detected by the anemometer.

The approach which we will use for evaluating Eq. (20) will consist of a sequential evaluation of first the integration with respect to time and then the spatial integration. This is done by first dividing the probe volume into b small slices of thickness $\Delta\eta$, as shown in Fig. 1. For each small slice, the time average velocity will be denoted as $\bar{U}_j$. In the limits as $\Delta\eta\rightarrow 0$

$$\bar{U}_j=(1/T)\int_0^T U(\eta_j,t)\,\mathrm{d}t \tag{21}$$

The evaluation of Eq. (21) for turbulent flow has been considered by McLaughlin and Tiederman.[3] This evaluation was based upon the postulate that when the seed particles are randomly distributed with respect to number density in the stationary fluid and when the number density is small, then the probability of detecting a seed particle in the probe volume is a function of the volume swept out by the velocity vector and the projected area of the probe volume. Based upon this concept, they showed that

$$(1/T)\int_0^T U(t)\,\mathrm{d}t = \sum_{k=1}^{a_j} \omega_{jk} U_{jk} \Big/ \sum_{k=1}^{a_j} \omega_{jk} \tag{22}$$

Here, a_j is the number of individual velocity realizations, U_{jk}, in the j^{th} slice of the probe volume, and ω_{jk} are weighting functions that are equal to $(|\mathbf{V}_{jk}|\gamma_{jk})^{-1}$ where V_{jk} is the instantaneous velocity vector in the probe volume and γ_{jk} is a geometric factor. Since the evaluation of this weighting function requires knowledge of all three velocity components, a simplified one-dimensional weighting function equal to $(U_{jk})^{-1}$ was proposed. Using the simplified weighting function, Eq. (21) becomes

$$\bar{U}_j = a_j \Big/ \sum_{k=1}^{a_j} (1/U_{jk}) \tag{23}$$

Since $\bar{U}_j$ is an accurate and properly weighted estimate of the time-average velocity in the j^{th} slice, the contributions to $\bar{U}_m$ from each slice are equally weighted. Thus

$$\bar{U}_m = (1/w)\int_{-w/2}^{w/2} \bar{U}(\eta)\,\mathrm{d}\eta = (1/b)\sum_{j=1}^{b} \bar{U}_j \tag{24}$$

and combining Eqs. (23) and (24)

$$\bar{U}_m = (1/b)\sum_{j=1}^{b} \Big[a_j \Big/ \sum_{k=1}^{a_j} (1/U_{jk})\Big] \tag{25}$$

When the probability of a realization is proportional to the instantaneous velocity, then a_j is proportional to $\bar{U}_j$ and the double summation in Eq. (25) is equivalent to

$$\bar{U}_m = N \Big/ \sum_{i=1}^{N} (1/U_i) = U_c \tag{26}$$

where N is the total number of realizations U_i from the entire probe volume. This is an important result because Eq. (26) provides a mechanism for evaluating $\bar{U}_m$ from the laser Doppler anemometer output.

However, the important remaining question is what is the time-average velocity $\bar{U}_0$ at $\eta = 0$? Namely, is $\bar{U}_0$ equal to $\bar{U}_m$? This question can be answered by evaluating Eq. (20) for a known or an assumed spatial variation of the time-average velocity. When the time-average velocity profile is linear, that is

$$\bar{U}(\eta) = (1/T)\int_0^T U(\eta,t)\,\mathrm{d}t = \bar{U}_0 + S\eta \tag{27}$$

Then Eq. (20) can be integrated to yield

$$\bar{U}_m = \bar{U}_0 \tag{28}$$

Thus, the weighted summation for a finite probe volume given by Eq. (26) yields a good estimate of the time-average mean velocity at the center of the probe volume when the time average fluid velocity profile is linear.

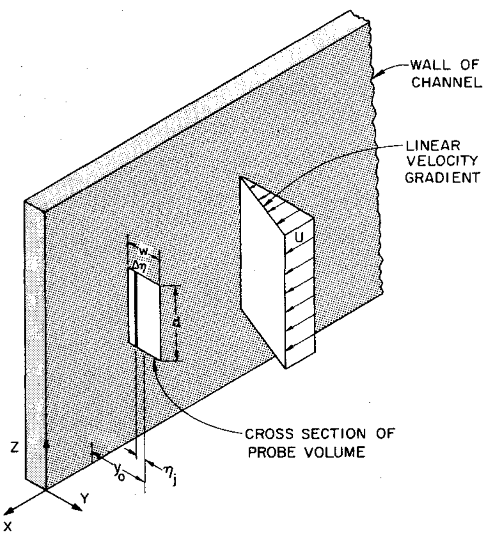

Fig. 1 Sketch of the mean flowfield and the cross section of the LDA probe volume.

Analysis of Root Mean Square Velocity Measurement

The measurement of fluid turbulence is often characterized by the root-mean-square velocity. The spatial and temporally averaged mean-square fluctuation of the streamwise fluid velocity flowing through a LDA probe volume is given by

$$u'^2_m = (1/A)\int\!\!\int [(1/T)\int_0^T (U(A,t) - \bar{U}_0)^2\,\mathrm{d}t]\,\mathrm{d}A \tag{29}$$

when the velocity fluctuations are referenced to the time-average velocity at the center of the probe volume. If the time-average velocity is only a function of y and if the probe volume is a rectangle (Fig. 1), then Eq. (29) becomes

$$u'^2_m = (1/w)\int_{-w/2}^{w/2} [(1/T)\int_0^T (U(\eta,t) - \bar{U}_0)^2\,\mathrm{d}t]\,\mathrm{d}\eta \tag{30}$$

The first task is to show how u'^2_m can be estimated from an ensemble of individual realizations. This can be accomplished in the same manner as the evaluation of $\bar{U}_m$ from LDA data. Again, after dividing the probe volume into b slices of thickness $\Delta\eta$, the time-average mean-square fluctuation of each slice u'^2_j can be evaluated using the one-dimensional weighting function proposed by McLaughlin and Tiederman.[3] That is

$$u'^2_{j0} = (1/T)\int_0^T [U(\eta_j,t) - \bar{U}_0]^2\,\mathrm{d}t \tag{31}$$

becomes

$$u'^2_{j0} = \sum_{k=1}^{a_j} (1/U_{jk})(U_{jk} - \bar{U}_0)^2 \Big/ \Big(\sum_{k=1}^{a_j} (1/U_{jk})\Big) \tag{32}$$

The double subscript "$j0$" is used to indicate that the mean-square fluctuation in the j^{th} slice is with reference to the time-average velocity at $\eta = 0$. Since the weighted summation given in Eq. (32) yields an accurate estimate of the time average mean-square fluctuation, the contribution to u'^2_m from each u'^2_{j0} are equally weighted. Thus

$$u'^2_m = \frac{1}{b}\sum_{j=1}^{b}\Big\{\sum_{k=1}^{a_j} (1/U_{jk})[U_{jk} - \bar{U}_0]^2 \Big/ \sum_{k=1}^{a_j} (1/U_{jk})\Big\} \tag{33}$$

And, when a_j is proportional to $\bar{U}_j$, Eq. (33) is equivalent to

$$u'^2_m = \sum_{i=1}^{N} (1/U_i)[U_i - \bar{U}_0]^2 \Big/ \sum_{i=1}^{N} (1/U_i) \tag{34}$$

The important point is that Eq. (34), which is a weighted estimate of the root-mean-square velocity, should be used to estimate u'_m from an ensemble of individual realization LDA data.

If a time-average flow field is either known or assumed, then Eq. (30) can be integrated and the effect of the finite-size probe volume can be evaluated. As before, the variation in the time-average velocity in space will be assumed to be linear [see Eq. (27)]. It is also necessary to make some assumption about the spatial variation of the time-average mean-square velocity fluctuation. To be consistent with our later use of this analysis for the correction of viscous sublayer data, it will be assumed that

$$u'(\eta) = \mathrm{K}\bar{U}(\eta) = \mathrm{K}(\bar{U}_0 + S\eta) \tag{35}$$

Thus

$$(1/T)\int_0^T [U(\eta,t) - \bar{U}(\eta)]^2 \mathrm{d}t = K^2(\bar{U}_0 + S\eta)^2 \tag{36}$$

Equation (30) can be rearranged so that the mean-square fluctuation at a point, calculated with the time-average velocity at the same point, $u'^2(\eta)$, appears explicitly. Namely

$$u'^2_m = (1/w)\int_{-w/2}^{w/2}\left[1/T\int_0^T (U(\eta,t) - \bar{U}(\eta))^2\mathrm{d}t + \{\bar{U}(\eta) - \bar{U}_0\}^2\right]\mathrm{d}\eta \tag{37}$$

Using Eqs. (35) and (36), Eq. (37) may be integrated to yield

$$u'^2_m = \mathrm{K}^2\bar{U}_0^2 + \frac{S^2w^2}{12} + \frac{S^2w^2\mathrm{K}^2}{12} \tag{38}$$

However, $\mathrm{K}^2\bar{U}_0^2$ is equal to the mean-square fluctuation at $\eta = 0$ and therefore

$$u'^2_m = u'^2_0 + \frac{S^2w^2}{12} + \frac{S^2w^2\mathrm{K}^2}{12} \tag{39}$$

Thus, the variance of the LDA ensemble originates from three sources: 1) u'^2_0 – the velocity variance caused by turbulence at the center of the probe volume: 2) $(S^2w^2)/12$ – the velocity variance caused by the change in mean velocity across the probe volume [note this term is the same as the "false turbulence" term in Eq. (18)]; 3) $(S^2w^2K^2)/12$ – the velocity variance caused by the change in the mean-square fluctuations across the probe volume.

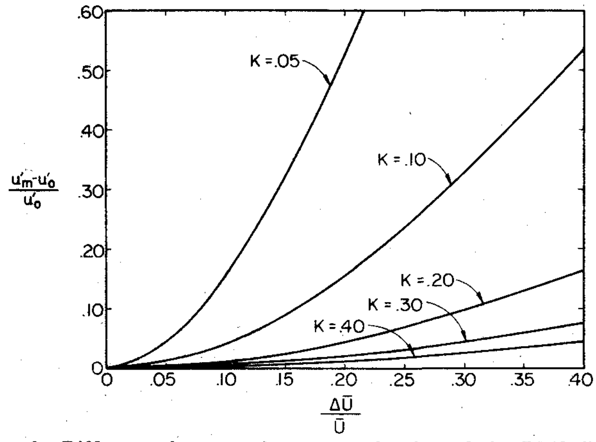

Fig. 2 Difference between the measured value of the RMS fluctuation and the value at the center of the probe volume.

Since the quantity of interest is usually the velocity variance at the center of the probe volume due to turbulence, Eq. (39) can be solved for it. Rearranging Eq. (39) and eliminating K with Eq. (35) yields

$$u_0'^2 = \left(u'^2_m - \frac{S^2w^2}{12}\right)\Big/\left(1 + \frac{S^2w^2}{12\bar{U}_0^2}\right) \tag{40}$$

Equation (40) can be used to calculate the root-mean-square velocity at the center of the probe volume when both u' and $\bar{U}$ vary linearly with y.

To illustrate the effect of the velocity gradient biasing, Eq. (38) can also be rearranged so that

$$\frac{u'_m - u'_0}{u'_0} = \left(\frac{\Delta\bar{U}^2}{12\mathrm{K}^2\bar{U}^2} + \frac{\Delta\bar{U}^2}{12\bar{U}^2} + 1\right)^{1/2} - 1 \tag{41}$$

where $\Delta\bar{U}$ is the change in mean velocity across the probe volume, Sw. The left-hand side of the equation is the fractional difference between the weighted root-mean-square of the measured velocity fluctuations and the root-mean-square of the velocity fluctuations at the center of the probe volume. It is plotted as a function of the nondimensional change in the mean velocity across the probe volume for several values of the turbulence parameter K in Fig. 2.

Two important effects can be seen from the figure. First, as the turbulence decreases, the percentage error in not correcting, for the velocity gradient biasing increases rapidly. This is because the "false turbulence" becomes the dominate part of the measured variance. The second is that as $\Delta\bar{U}/\bar{U}$ increases, as near the wall in the viscous sublayer, the error increases.

Table 1 Summary of LDA measurements in the viscous sublayer

Reynolds number	y (in.)	U_B (fps)	U_c (fps)	% uncertainty in mean	u'_B (fps)	u'_m (fps)	u'_0 (fps)	% uncertainty in rms	N
7,470	0.0015	0.0666	0.0521	±11.2	0.0363	0.0275	0.0184	+26.9 −17.5	85
7,470	0.0033	0.1079	0.0915	±3.8	0.0465	0.0387	0.0328	+9.9 −8.2	474
7,470	0.0052	0.1692	0.1474	±2.6	0.0639	0.0567	0.0529	+7.3 −6.4	826
7,470	0.0091	0.2968	0.2587	±1.8	0.1088	0.0993	0.0971	+4.9 −4.5	1737
7,470	0.0130	0.4182	0.3688	±1.5	0.1411	0.1350	0.1334	+4.2 −3.9	2375
12,790	0.0021	0.1853	0.1551	±4.9	0.0779	0.0684	0.0470	+12.4 −10.1	309
12,790	0.0041	0.3380	0.2949	±2.8	0.1285	0.1128	0.1009	+8.0 −7.0	696
12,790	0.0061	0.4906	0.4317	±1.7	0.1752	0.1595	0.1513	+5.8 −5.2	1273

The use of Eq. (40) to correct LDA data implies a certain amount of knowledge about the flowfield. In some cases it may be more appropriate to use the results of this analysis in the form of Eq. (41) to define a maximum size probe volume for a maximum allowable error.

Application of Corrections to Viscous Sublayer Data

The result of using the proposed corrections will now be evaluated using individual realization LDA data that was taken in the viscous sublayer of a turbulent channel flow of water. The data was taken with the side-scatter optical configuration described by Karpuk and Tiederman.[6] In this configuration the incident laser beams were parallel to the channel walls and the axis of the receiving optics was along a line perpendicular to the plane of the laser beams. The receiving optics viewed only the center portion of an elongated ellipsoidal probe volume so that the effective cross section was a rectangle. The dimension of the probe volume in the direction of the velocity gradient was reduced to 0.00235 in. with a one-dimensional beam expander, while the thickness of the viscous sublayer was on the order of 0.01 in. The LDA used a "common-mode" rejection, optical technique[9,10] to insure sufficient bandwidth for the highly turbulent sublayer.

The water was filtered to remove all particles larger than 0.5 μm and then 5 – 10 μm sand was added to provide scattering particles. the results of Hjelmfelt and Mockros[11] were used to verify that these scattering particles would accurately follow the velocity fluctuations. The seed was sufficiently dilute so that a scattering particle was in the probe volume only about 1.6% of the time. Consequently, the individual realizations are independent.

The sublayer data for steady flow at two Reynolds numbers are shown in Table 1. These data were reduced using a sequential phase comparator[12] to insure that each velocity realization was accurate. A method such as this for validating each realization is necessary because it is not unusual for a Doppler burst to either have cycles missing or suppressed. Estimates of the 95% confidence intervals for the mean and root-mean-square velocities were made using the statiscal approach suggested by Yanta and Smith.[13] Notice that the ensembles are relatively large and that all but one estimate of the mean velocity are accurate to ±5% or better.

The biased and corrected estimates of the mean velocity are plotted in Fig. 3. The main features to notice are the linearity of the data and the 10% difference between the corrected and biased estimates of $\bar{U}$. In contrast to Kried's suggestion,[4] it is possible to make sufficiently accurate LDA measurements that a biasing correction will have a significant effect upon the results.

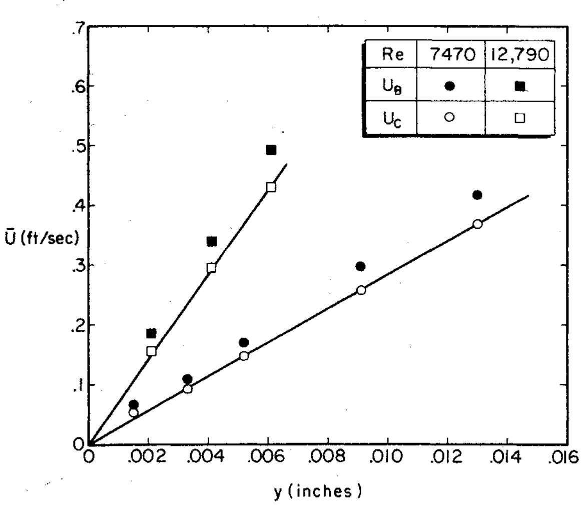

Fig. 3 Biased and corrected estimates of the mean velocity in the viscous sublayer of channel flows.

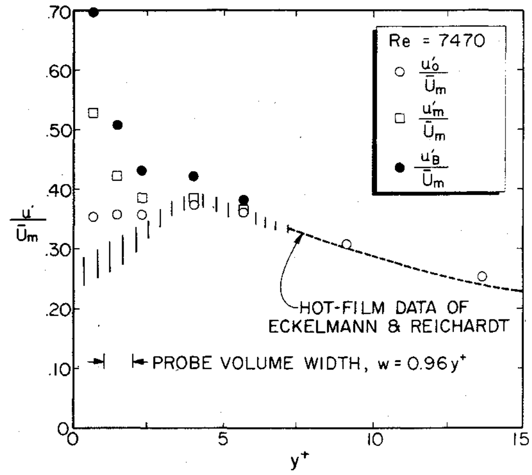

Fig. 4 Biased and corrected estimates of the RMS velocity, normalized with the local mean velocity, for a channel flow at a Reynolds number of 7470.

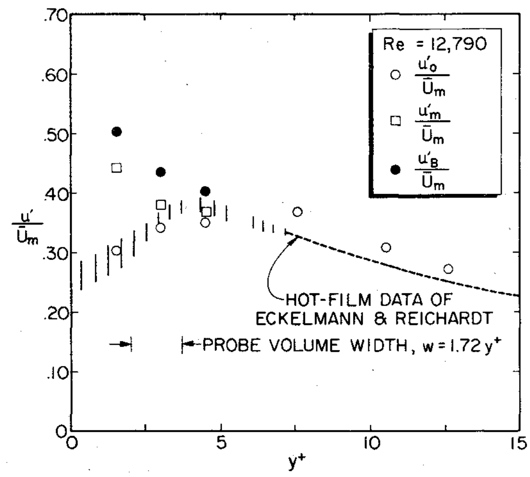

Fig. 5 Biased and corrected estimates of the RMS velocity, normalized with the local mean velocity, for a channel flow at a Reynolds number of 12,790.

Three separate estimates of the root-mean-square of the velocity fluctuations within the viscous sublayer are plotted in Figs. 4 and 5 as a function of nondimensional wall layer coordinate

$$y^+ = (yU_\tau)/\nu \tag{42}$$

The highestes estimates are given by an unweighted and uncorrected calculation where

$$(u'_B)^2 = \frac{1}{N-1}\sum_{i=1}^{N}(U_i - U_B)^2 \tag{43}$$

The middle estimates were made using Eq. (34). These weighted estimates are still high because the contributions from the spatial variation of $\bar{U}(y)$ and $u'(y)$ within the probe volume have not yet been removed. The best and lowest estimates are given by Eq. (40), which yields estimates that are completely corrected for biasing due to both temporal and spatial variations in the velocity field.

Also shown in Figs. 4 and 5 are LDA data from locations outside the viscous sublayer[6] and the hot-film measurements of Eckelmann and Reichardt.[8] The uncorrected estimates from the LDA for $y^+ < 5$ are clearly too high. However, the proposed correction brings the LDA data into excellent agreement with the hot-film data. Note also how the biasing decreases as y^+ increases. This is expected because $\Delta\bar{U}/\bar{U}$ is decreasing rapidly as y^+ increases.

Summary and Conclusions

The important conclusions are: 1) The weighted average of an ensemble of LDA data yields a good estimate of the spatial and temporal average of the fluid velocity through a finite probe volume [see Eq. (26)]. 2) For a linear velocity gradient, this weighted average is also a good estimate of the time-average velocity at the center of the probe volume in both laminar and turbulent flows [see Eqs. (17) and (28)]. 3) The weighted estimate of the mean-square fluctuations of an ensemble of LDA data yields a good estimate of the spatial and temporal average of the mean-square of the fluid velocity fluctuation [see Eq. (34)]. 4) The mean-square of the fluid velocity fluctuations within a finite probe volume consists of a contribution from turbulent fluctuations at the center of the probe volume, a contribution from the spatial variation of the time-average fluid velocity, $\bar{U}$, and a contribution from the spatial variation of the root-mean-square fluid velocity, u' [see Eq. (39)]. 5) When the spatial variation in $\bar{U}$ and u' are known, or when reasonable estimates can be made, then accurate estimates of the mean and root-mean-square velocity at the center of the probe volume can be made from an ensemble of LDA data. Equations (26) and (40) yield good estimates for a rectangular probe volume in a flow where $\bar{U}$ and u' vary linearally. 6) In the near-wall region of a turbulent flow, biased estimates of u' made from an ensemble of LDA realizations can be as much as 100% too high while biased estimates of $\bar{U}$ are about 10% high. Whenever the velocity fluctuations are high, and/or whenever the velocity gradient across the probe volume is high, then the bias due to both the spatial and temporal variations must be removed before accurate estimates of $\bar{U}$ and u' can be made.

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